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Draft Progress Report

Phase I

# ENERGY EFFECTIVENESS OF ARRAYS OF WIND ENERGY COLLECTION SYSTEMS

Prepared for

Mr. Sven Hugosson
National Swedish Board for Energy Source Development
Box 21048
S-11428 Stockholm, Sweden

3 January 1977

AEROVIRONMENT INC.
145 VISTA AVE. PASADENA, CALIFORNIA 91107
(213) 449-4392

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# ENERGY EFFECTIVENESS OF ARRAYS OF WIND ENERGY COLLECTION SYSTEMS

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Mr. Sven Hugosson
National Swedish Board for Energy Source Development
Box 21048
S-11428 Stockholm, Sweden

By

Peter B.S. Lissaman

AeroVironment Inc.
145 Vista Avenue
Pasadena, California 91107
U.S.A.

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## 1. SUMMARY OF PROGRESS

The initial analytical studies toward developing models for an arbitrary array of WECS have been completed. Computer programs have been composed and operated for a number of cases, and power isopleths constructed for a circular receptor rotor of arbitrary size and position in the wake of single rotor of arbitrary size and power. These programs are up and running for the following cases:

1) Non-turbulent unbounded outer flow

2) Unbounded outer flow of arbitrary turbulence

3) Rotor at arbitrary height above ground in unbounded outer flow of arbitrary turbulence.

No difficulties in constructing the models have been experienced, and the power isopleths for the above cases are of sufficient character to be of direct operational utility. The extensive numerical studies have lead to considerable simplification in the mathematical formulations by identifying the significance or sensitivity of the various terms.

The directly operational conclusions evident so far are that the ambient turbulence is the dominant factor in wake reenergization and that the rotor and tower generated turbulence is of minor significance. This appears of great importance since it suggests that the aerodynamic details of the rotor and tower structure and rotor viscous power losses are of secondary importance in wake reenergization. Thus representative power isopleths for many different types of rotors can be derived from a few basic curves.

From the point of view of Phase II of the program, it appears that the development of a model for a full general rotor array will proceed in an orderly fashion, and that there will be no serious fluid mechanical complexities. The main effort in developing the full program will involve the details of proper and efficient treatment of the complex geometrical relationship involved in WECS arrays.

The results presented check properly against all known limit cases, both theoretical and experimental. However, no actual WECS field data is available for checking of an actual windmill wake case.

In general, the progress made in this portion of the work has been somewhat greater than anticipated, because the detailed numerical analysis has shown that some aspects of the wake development were simpler than had originally been expected.

## 2. DISCUSSION OF THE MECHANISMS OF WAKE DEVELOPMENT

### 2.1 General

The wake of a single power generating rotor may be expected to be quite similar fluid mechanically to that of a circular jet immersed in a uniform flow, where the initial velocity of the jet is uniform and lower than that of the outer flow. We use the word "jet," although this is usually associated with a flow of higher speed than the outer flow. In all cases here, the jet is actually wake-like, that is, its speed is lower than that of the outer flow. The situation is as sketched in Figure 1 and we can distinguish two regions (the initial and the fully developed regions) in which locally self-similar profiles obtain. Between these regions is a transition zone, in which the flow profile changes from one self-similar profile to another.

The general process of wake development is that the wake grows by turbulent entrainment at its edge, which introduces free stream momentum as well as mass into the wake, so these quantities are not conserved in the wake. However, the flow force, or drag, of the wake is conserved (at least for an unbounded outer flow) so that for a constant pressure wake the momentum deficit is conserved. This is defined by the integral of $u/U\,(1 - u/U)$ across the wake, where u is the wake velocity which is spatially variable, while U is the outer flow speed, which we will assume constant at this point. In a later section we discuss methods of incorporating a variable U.

Thus it is clear that this integral expression, D, where $D = \int u/U\,(1 - u/U)\,r dr$ is one of the controlling parameters in the wake description, the others are the wake growth rate dR/dx, and the wake profile at a radius r, given by $u/U = F\,(r/R)$ where R is the effective outer radius of the wake. The value of the wake drag can be computed from the initial state of the wake, which can be directly related to the drag of the rotor and the power output.

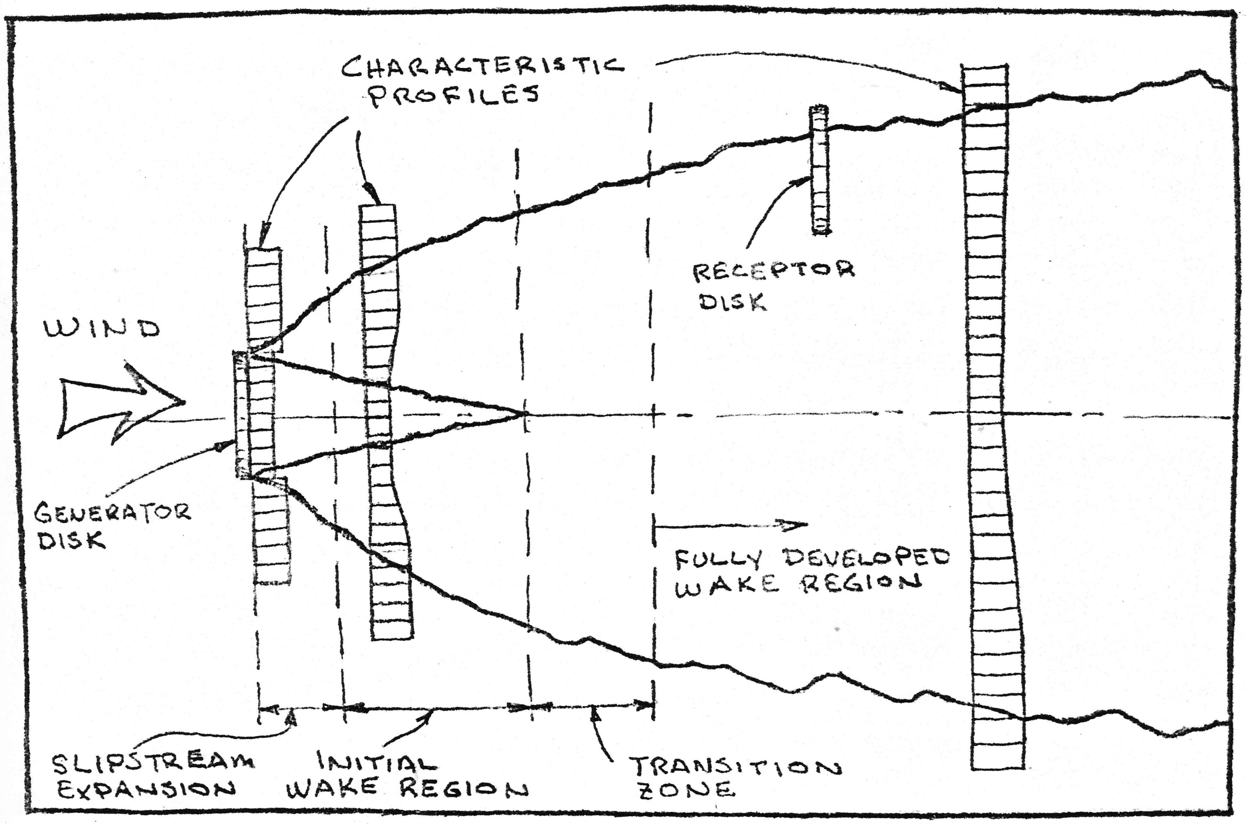


FIGURE 1. General regions of WECS wake in unbounded uniform flow.

Now, the turbulence near the edge of the wake controls its lateral growth. In general, this turbulence is some functional combination of that already in the flow (ambient turbulence), that generated by the shear due to the momentum gradients in the wake itself (momentum generated turbulence) as well as that generated by the viscous effects due to the rotor (rotor generated turbulence). Each of these terms has a different relative importance in different regions of the wake.

The general process is as follows, in the initial region, strong turbulence is generated by the shear layer and general turbulence associated with power extraction. This turbulent region extends outward and inward from the edge of the wake. The initial region terminates when these shear layers meet at the center, so that the center velocity deficit starts to reduce. It may be expected that the rotor viscous generated turbulence will, to a large extent, have dissipated at this point.

In the main jet region, the distributed shear generates turbulence. This turbulence generation is proportional to the centerline velocity deficit divided by the effective wake radius. The momentum turbulence steadily decays, so that the wake growth rate due to this effect steadily reduces. However, if the system is immersed in an ambient turbulence field, which is approximately isotropic and uniform, then we expect that this ambient turbulence will start to control the wake growth as the rotor and momentum turbulence decays. Thus, in the broadest sense, we expect the initial region growth to be controlled by momentum and rotor turbulence, and the main jet growth to be dominated by ambient turbulence. The magnitudes and roles of these turbulence mechanisms will be discussed in more detail later.

The actual profiles in the two regions are defined by the magnitude of the Reynolds stresses and the appropriate Navier-Stokes equations. Here we shall assume, following Abramovitch (1963a), that in the intial region the profile is given by a characteristic shear layer profile in the mixed zone outside the core, $r > r_c$; with a uniform profile in the core $r < r_c$. Thus

$$u/U = 1/m \qquad 0 < r < r_c$$

$$u/U = 1/m + \frac{m-1}{m} (1 - \eta^{1.5})^2 \qquad r_c < r < R$$

where $\eta = (r - r_c)/(R - r_c)$
where $R - r_c$ is the width of the turbulent mixed zone.

In the main jet region we again use the result of Abramovitch,

$$u/U = 1 - \Delta U (1 - \xi^{1.5})^2 \qquad 0 < r < R$$

where $\Delta U$ is the normalized velocity deficit and $\xi = r/R$. These profiles, which are well substantiated by numerous experiments, provide the basis from which we can compute the velocity profiles for any streamwise station in the flow, and hence the momentum deficit and other invariant features of the flow. Thus, if we assume the self-similar profiles in the two major regions the flow field is fully characterized by the velocity and radius scale, so it is only necessary to know the rate of wake growth to fully define the flow field.

Other factors enter into the problem if we consider that the outer flow field is non-uniform, as is certainly the case. This non-uniformity is introduced by the planetary boundary layer (and the ground plane) which causes the mean and turbulent velocity profiles to vary with height, and by the presence of wakes of nearby rotor systems. These effects are discussed in later sections.

## 2.2 Wake Growth

The growth and development of the wake is controlled by the turbulence and mean wake velocity gradient near its edge. It is not our goal here to enter into a detailed description of the turbulent structure, but simply to develop an approach which is functionally and dimensionally correct – that is, involves all the proper significant variables, and then to use scaling constants obtained from experiment.

Initially, we will consider the two limit cases; first that of a turbulent wake in a non-turbulent outer flow. In this case, turbulence is generated only by mechanical shear within the wake and is thus connected to the velocity deficit and the wake radius. If we take a dimensional approach we see that the temporal rate of wake growth, dR/dt must be related to $\Delta U$, the centerline velocity deficit, so that the spatial growth rate can be expressed

$$dR/dx \sim \Delta U$$

Noting, now that $\Delta U$ is connected to the wake radius by the invariant momentum deficit, we see that providing $\Delta U$ is not too large, $\Delta UR^2$ is constant so obtain

$$R^3 \sim x.$$

This result has been given in similar form by Schlichting (1962) who showed that it could also be derived by mixing length arguments.

The above case can be considered that of wake growth due to momentum generated turbulence, which we will define as (dR/dx)m.

The other limit case is that of the growth of the plume of an entirely passive contaminant. This is the situation which occurs in plume concentration for air quality modeling. Now it can be shown here that for short-range dispersion the plume dimension in the planetary boundary layer grows approximately proportionately to the ambient turbulence (Lissaman, 1973) so that we can write the ambient growth rate as

$$(dR/dx)_a \sim \alpha$$

where $\alpha$ is a spatial constant but depends upon the environmental conditions of wind speed, ground roughness and insolation. For terrain of about 1 cm roughness an equivalent can be determined for each Pasquill stability class, (Pasquill, 1962) while an algorithm for determining $\alpha$ for different ground roughness has been proposed by Lissaman (1973).

Thus we see that there are two basic turbulence terms: one due to the momentum turbulence, which dominates in the early stages but decays quite rapidly with downstream distance, and one due to the ambient turbulence which dominates in the far wake. A method of estimating growth due to the combined effects of mechanical and ambient turbulence is discussed in a following section.

## 2.3 Wake Development in a Non-Turbulent Infinite Medium

This case has been quite extensively studied (Abramovitch, 1963). In the cited reference extensive experimental data for jets of higher and lower speed than the outer flow are given. A parameter m is defined as the ratio of the outer flow velocity U, to that of the initial jet $U_o$, $m = U/U_o$. For all our cases $m > 1$. Abramovitch proposes normalized profile functions and wake growth laws with universal constants appropriate to our purposes. The growth rates provide significant data to determine the relative magnitude of growth due to atmospheric turbulence.

The case of a WECS wake developing in a non-turbulent atmosphere was developed and run as a basic case for comparison purposes. Power isopleths are presented and discussed in a later section.

## 2.4 Wake Development in a Turbulent Infinite Medium

If the wake is immersed in a turbulent medium, then both ambient and momentum generated turbulence contribute to the growth rate. As has been already noted, the case for a wake with momentum in a non-turbulent medium has been well-documented, as has that for a plume in a turbulent flow, which can be regarded as a 'wake' with vanishing momentum. What we require is a rational technique for defining wake growth where both momentum turbulence and ambient turbulence are present.

The approach is to assume that the wake growth at the edge of the wake is proportional to the rms turbulent velocity in that region, regardless of how this turbulence has been generated. Thus we assume $dR/dx \sim V/U$ where V is the turbulent velocity at the wake edge. Now, if we consider the generation of this turbulence due to the momentum alone we note that the general process is that the velocity shear removes kinetic energy from the mean flow and generates turbulent energy at a number of scales related to the wake width. Simultaneously, energy is continuously dissipated in the microscale range, and the energy difference between that removed from the mean flow and that dissipated appears as turbulent energy. An analogous process occurs with the ambient turbulence, where the planetary boundary layer shear serves as the generating mechanism, with a similar small-scale dissipation.

If we assume both these processes to occur independently, as seems likely, because of the large difference in scales of the generative process, then it is plausible to argue that the total turbulent energy at any station due to both processes occurring simultaneously is simply the source of the turbulent energy generated in each case. On this basis we write

$$V_T^2 = V_{Tm}^2 + V_{Ta}^2.$$

where $V_T$ is the total turbulent velocity near the wake edge and $V_{Tm}$, $V_{Ta}$ that due to momentum and ambient effects respectively. Our argument now is that $V_T$ controls the total wake growth, or $dR/dx \sim V_T/U$.

This establishes the basic functional addition law for the two turbulent growths, and we can thus write the total growth rate as

$$dR/dx = \sqrt{(dR/dx)^2{}_m + (dR/dx)^2{}_a}$$

where the terms on the right hand side represent the growth due to purely momentum effects and that due to ambient turbulence.

It is of some interest to consider the magnitudes of the various terms. The ambient term can be shown to be given by

$$(dR/dx)_a = \alpha/.51$$

where $\alpha$ is the growth rate of mean width of a plume in a turbulent atmosphere, and the numerical constant is obtained by equating the scalar transport of the Gaussian distribution used in plume analysis to the particular near Gaussian distribution used here. This growth rate is approximately constant with distance, and for a representative terrain, wind speed and atmospheric stability $\alpha = .05$. We note that $\alpha$ varies with the above environmental parameters and for numerical models the appropriate $\alpha$ will be used.

The momentum growth rate on the other hand, is strongly dependent upon downstream distance, decaying very rapidly as the wake velocity deficits get smaller. For the main jet region Abromovitch gives this rate as

$$(dR/dx)_m = 0.22\ (1 + 2m/\Delta U\ (1-m))^{-1}$$

We note that at the start of the main jet region we get (for m = 2)

$$(dR/dx)_m = .070$$

while about 30 radii downstream in a turbulent outer flow this term has dropped to

$$(dR/dx)_m = .006.$$

For $\alpha = .05$, the constant ambient growth rate is about

$$(dR/dx)_a = .10.$$

Now $\alpha$ will seldom have a value of less than .05, because such values occur only in very light winds with strongly stable conditions, when the wind energy systems would be unlikely to be in operation. Consequently, we see a very important simplifying factor, that for most of its development the wake growth is controlled by ambient turbulence and not by momentum generated turbulence.

It is clear that during the initial growth period, both terms are of about the same order. To investigate the magnitude of this combining effect, the total growth rate expression was integrated numerically for a wide range of different cases. It was found that the ambient terms were in fact dominant very early in the growth process. Plots of the exact growth equation are shown in the results section.

## 2.5 Effect of Neighboring Wakes

For two rotor systems side by side, the wakes will eventually interact and coalesce. Now evidently in this case the momentum generated turbulence will be affected by the interaction of the two momentum wakes, and by the momentum transport between them. However, it has been shown in the previous section that after a few tens of radii downstream the momentum generated turbulence is negligible compared to the ambient terms. Thus, the transfer will be similar to that of an inert scalar. To restate this point, we note that, although we are considering transfer of momentum by its own and ambient turbulence, because the momentum turbulence is so weak compared with the ambient turbulence we can consider the transfer as though the momentum (or velocity deficit) were a passive scalar.

Thus the situation becomes analogous to pollution concentration due to neighboring plumes where, because of the linearity of the process one may simply superimpose the respective plume concentrations. This superposition conserves pollutant mass in the analog situation and in our case conserves the linearized momentum deficit and thus satisfies drag conservation.

Fortunately, it is unlikely that the geometrical arrangement of a WECS array will be such that wakes of neighboring systems interfere within such short distances that momentum interaction need be considered in estimating turbulent growth.

## 2.6 Effect of the Ground Plane

The ground plane has an effect in reducing vertical downward wake growth, both by the suppression of turbulent velocity near the ground and by the direct presence of the ground (the inviscid potential effect). For the concentration of inert pollutants the ground effect has been extensively studied, and the standard approach is to model it by classical reflection techniques. This model does not directly account for the suppression of turbulence near the ground (however, this suppression occurs only in a region very close to the ground) but gives rational ground effect profiles, conserving pollutant mass flow and exhibiting zero vertical transfer rates at the ground. These models have been well-substantiated in air quality modeling.

Now we have already established that a short distance from the rotor, the momentum wake may be regarded as a passive scalar under most representative conditions of ambient turbulence. Thus we may model the ground plane effect by imaging in the conventional way. This technique will preserve wake momentum deficit and assure no wake momentum transfer into the ground.

It is of some theoretical interest here to note that according to these arguments, the vertical dividing plane between two adjacent identical rotors can be treated exactly like the ground plane of a single rotor. The momentum transfer, however, appears to be physically different. In neither case is there any net momentum transfer across the dividing plane. For the line abreast pair we note that ambient turbulence is clearly present near the dividing plane, and there is, in fact, significant momentum flux across this plane. But, because of the symmetry, the momentum flow from left to right is equal to that in the other direction and the net momentum flux is zero. In the case of the ground plane, of course, there is no momentum flux to or from the ground, so that here again the net momentum flux is zero.

By continuing these arguments it is clear that a row of rotors in ground effect may be modeled by superposition of the rotor wake and their images.

These arguments are subject to the same conditions of validity as mentioned previously, that the rotors be neither too close to the ground nor to each other.

# 3. DEVELOPMENT OF ANALYTICAL MODEL

## 3.1 General

Analytical expressions for wake geometry and velocity profiles have been developed by Abramovitch (1963) for the case of a momentum deficit wake in an unbounded, external flow with zero ambient turbulence. As a starting point for the development of the present model, these expressions were formulated for solution on a digital computer (Hewlett Packard 9820A) with the variables of interest as input. These included size and power extraction of the upwind disk and size and location of the downwind, or receptor disk.

The output from the computer code included the wake radius and kinetic energy flux (referenced to free stream conditions) at any specified downstream location.

Included in the logic of the code was a routine for integration of wake power over a circular disk of arbitrary diameter, centered at a given downwind location. Thus, the variation of the power profile across the wake could be taken into account as the flux of kinetic energy through the downstream receptor disk. A discussion of the integration method follows later.

The general results of this form of the model are presented in Figure 2. Lines of constant power are shown with respect to downstream wake geometry for a receptor disk diameter equal to the power extraction disk diameter. That is, these are power isopleths for an identical rotor pair. These values of power can be regarded as the ratio of energy flux across the receptor disk to that available if the upstream rotor were not present. Cross-flow wake dimensions are shown greatly distorted in scale with respect to downwind wake dimensions on this diagram. The figure represents a preliminary output from the code, and indicates the relatively slow wake reenergization in a non-turbulent outer flow. This figure is drawn

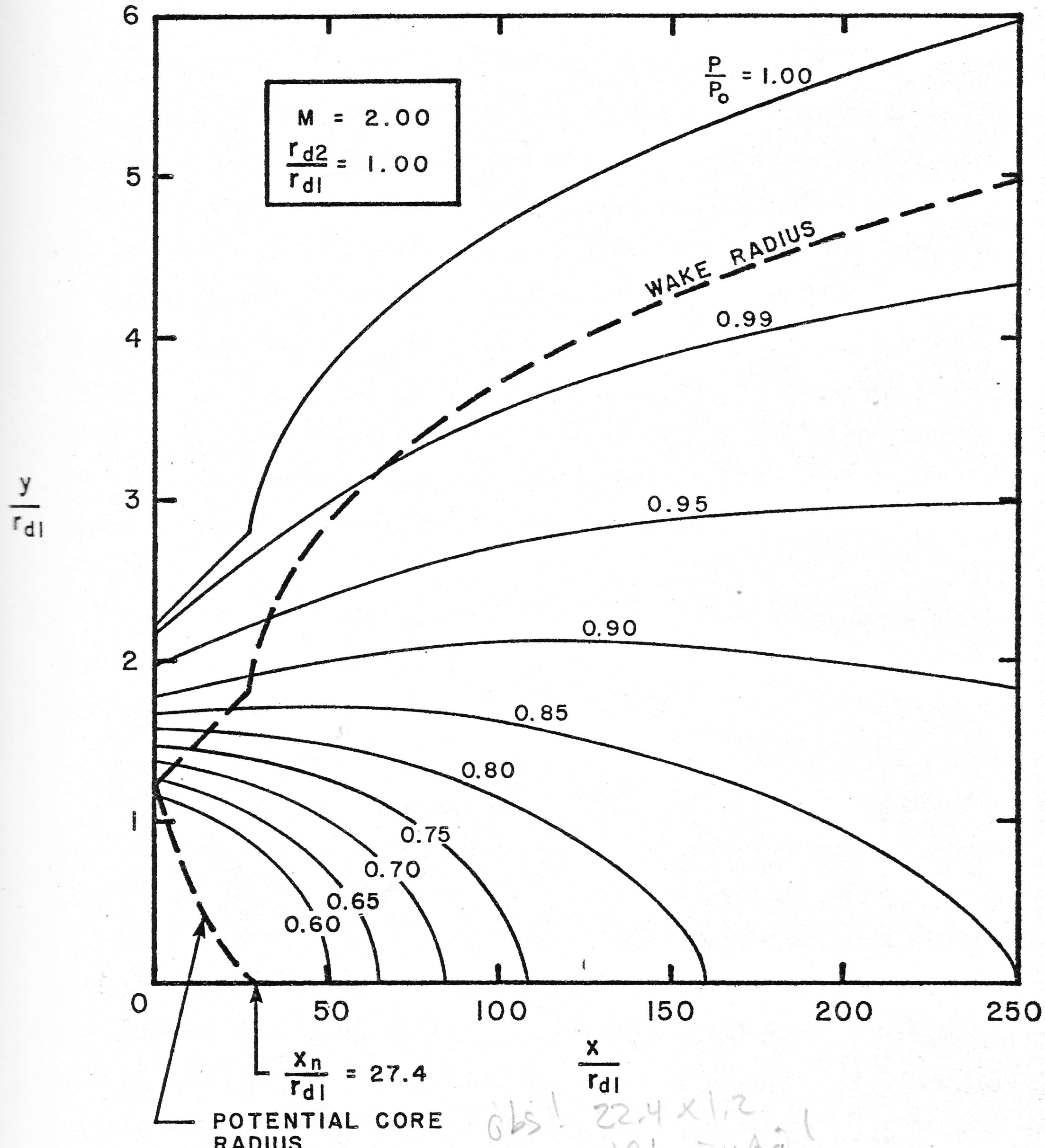


FIGURE 2. Isopleths for non-turbulent unbounded outer flow (generator disc $C_p = 0.50$, receptor disc radius $r_{d2}$ equal to generator, $r_{d1}$).

for the upstream, or generator, rotor operating at a power coefficient of 0.50, corresponding to m = 2.00.

The code was then modified to include the effects of ambient turbulence in the external flow. The model was revised to reflect a wake growth rate determined by the combined effects of mechanical and ambient turbulence. The resulting increased wake growth produced reductions in the velocity deficit, and a corresponding increased amount of available power for the same downstream location compared with the zero ambient turbulence case. The required additional input to the code is the turbulent dispersion for the level of turbulence being investigated. Figure 3 shows the isopleths for the effects of ambient turbulence for the same geometry as shown in Figure 2 and for the turbulent dispersion, $\alpha$ . This value of $\alpha$ is representative of the natural turbulent for wind of about 4 m/s over flat terrain of about 1 cm roughness in neutral stability.

Geometrical features associated with wake generation and development have been normalized with respect to the diameter of the upstream disk in both of the figures.

The effects of the ground plane were then investigated and the code was modified to incorporate the height above the ground as an input to the calculations for the power profiles in the wake. A standard image technique was used, and the code was changed to appropriately account for the wake velocities from the main source as well as its image during the integration process over the receptor disk. This modification produced typical power isopleths of quite similar form to those of Figure 3 except that the suppression of mixing due to the ground caused somewhat retarded wake reenergization. The effects are shown more clearly in tabular form in Table 3-1, which shows power ratios in the wake for the same conditions as presented in Figures 2 and 3, with addition of the case for a ground plane one rotor diameter from the rotor axis. Several downstream locations were selected for display in the table and for comparison, the power ratios from Figure 2 and Figure 3 for the same downstream distances are shown.

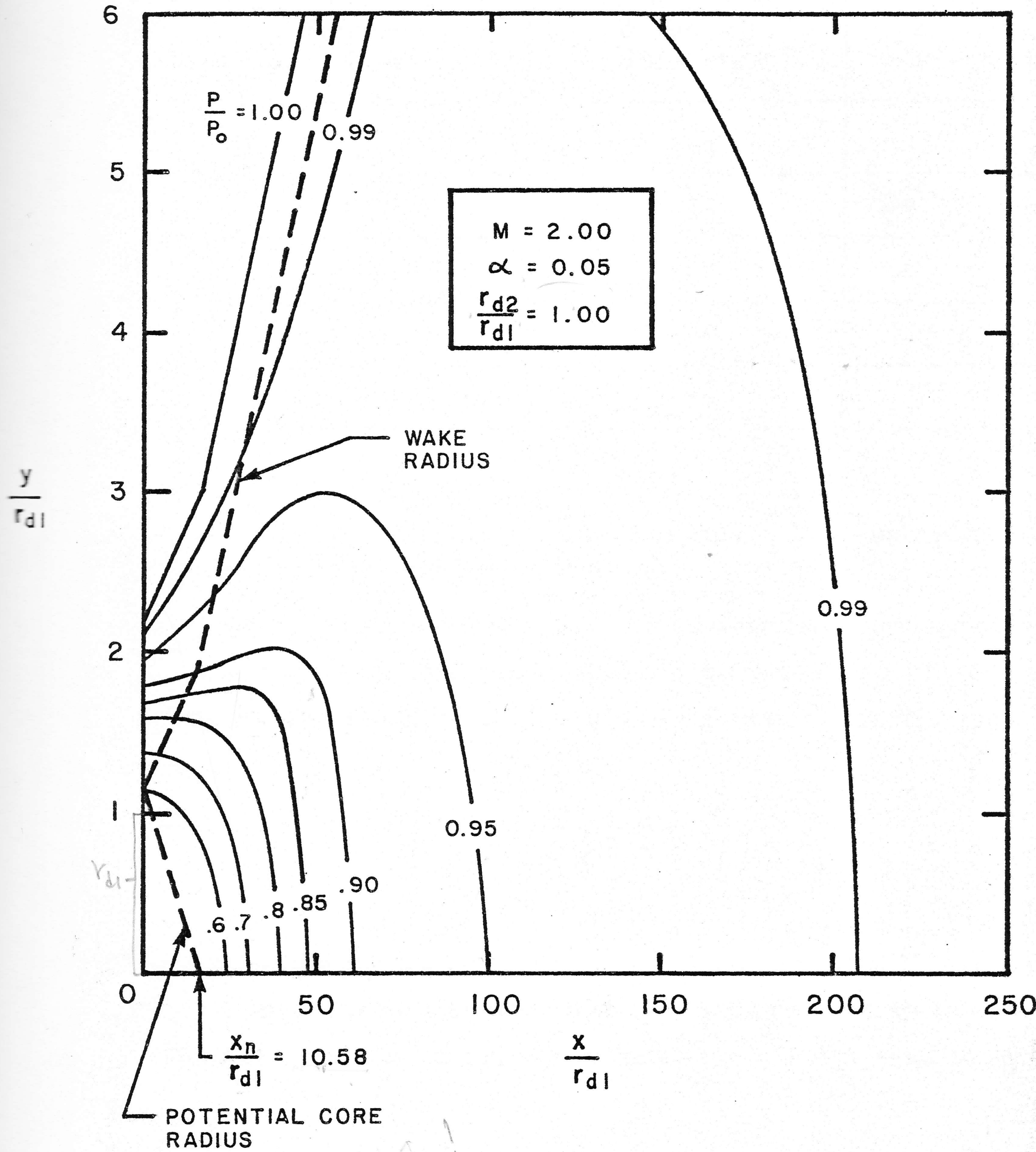


FIGURE 3. Power isopleths for turbulent unbounded outerflow (generator disc $C_p = 0.50$, receptor disc radius $r_{d2}$ equal to generator, $r_{d1}$).

TABLE 3-1.

| $\frac{X}{rd_1}$ | Centerline Power Ratios | | |
|---|---|---|---|
| | 1. Mechanical Turbulence Only | 2. Mechanical & Ambient Turbulence | 3. Same as 2, with Ground Plane, $h_0$=2$rd_1$ |
| 0 | .13 | .13 | .13 |
| 25 | .29 | .63 | .63 |
| 50 | .60 | .87 | .85 |
| 100 | .74 | .96 | .94 |
| 150 | .79 | .98 | .97 |

It will be seen that the ground effect is small, but unfavorable, in the sense that is delays wake reenergization.

The increase in the size of the newly formed wake from the size of the extraction disk due to normal slipstream expansion can also be seen in Figures 2 and 3. The radius after slipsteam expansion, $r_0$, at this point is given by

$$r_0/rd_1 = \sqrt{\frac{m+1}{2}}$$

It was assumed for the purposes of development of the code that this process takes place in a downstream distance which is small compared to the other scales of interest. For this reason, the wake radius was set equal to $r_0$ at the origin.

## 3.2 Wake Geometry and Velocity Profiles

Figure 4 shows the overall wake geometry and velocity profiles and in particular, the outer radius associated with wake growth. Figure 5 shows the wake radius regions and parameters used in the analysis.

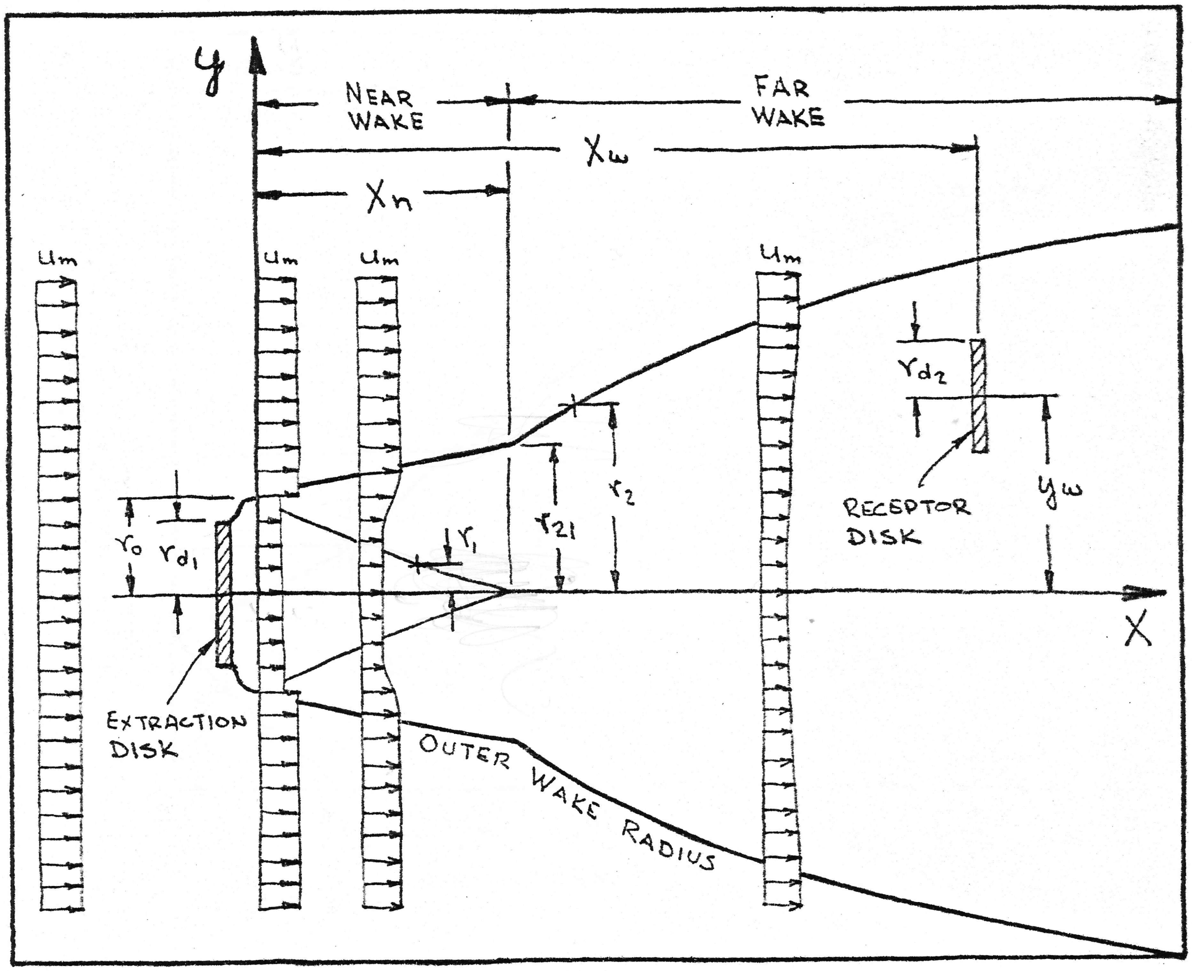


FIGURE 4. Wake geometry and velocity profiles.

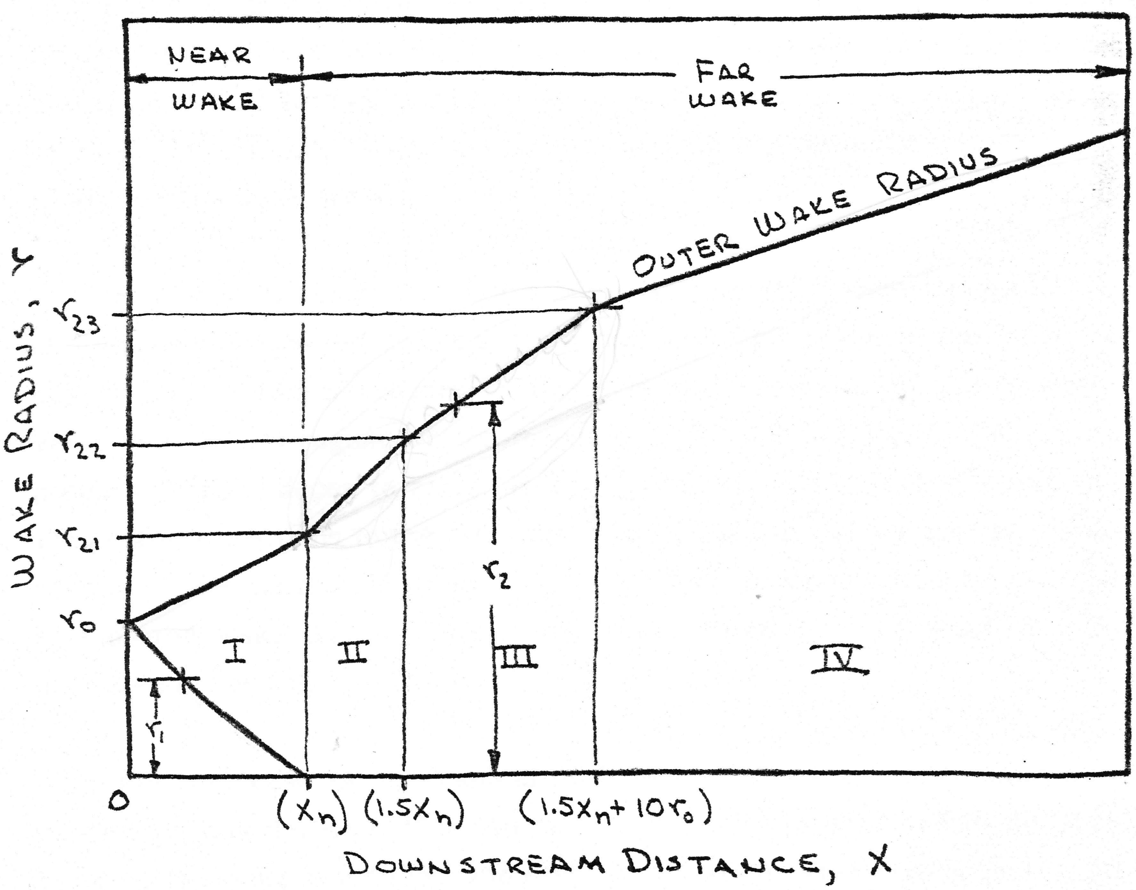


FIGURE 5. Wake growth regions.

The wake is divided into two major regimes, the near wake and the far wake. The significant difference between the two regimes from the standpoint of the analytical model is the form of the velocity profile. Figure 6 shows the shapes of the self-similar velocity profiles and the associated analytical formulations that were used to describe them in each wake regime. In addition, the properties of the constant velocity profile for a newly formed wake are also shown.

As the flow proceeds downstream, the constant velocity profile is continuously eroded due to shear from the outer flow. Hence the inner, or potential core radius reduces in size. In this region, both the constant velocity and shear regions are represented in the velocity profile. When the potential core reduces to zero, (at $x = x_n$) the wake is fully developed, and the profile transitions to the far wake velocity profile. This latter profile is scaled by the centerline velocity deficit, which itself is a function of the size of the wake due to considerations of conservation of total drag as discussed in Section 2.

For the case of mechanical turbulence only, a compact analytical expression was developed for $\Delta \bar{u}\, m$, connecting it to downstream distance by using the wake growth law and drag conservation. This expression for $\Delta \bar{u}$ m is a function of downwind distance, x, and initial velocity ratio, m, and is given by:

$$\Delta \bar{u}\, m = \frac{2.61}{(m - 1.03)^{.52} \left(x - x_n + .52 + \dfrac{4.92}{m - 1.01}\right)^{.59}} \qquad (1)$$

The form and the constants of the above expression were obtained from the complicated Abramovitch (1963) form (Abramovitch, Eqn. 5.106, p. 200) by transformation and curve-fitting techniques.

When ambient turbulence is present the growth of the wake, and hence $\Delta$um, are influenced significantly by the level of this turbulence and the

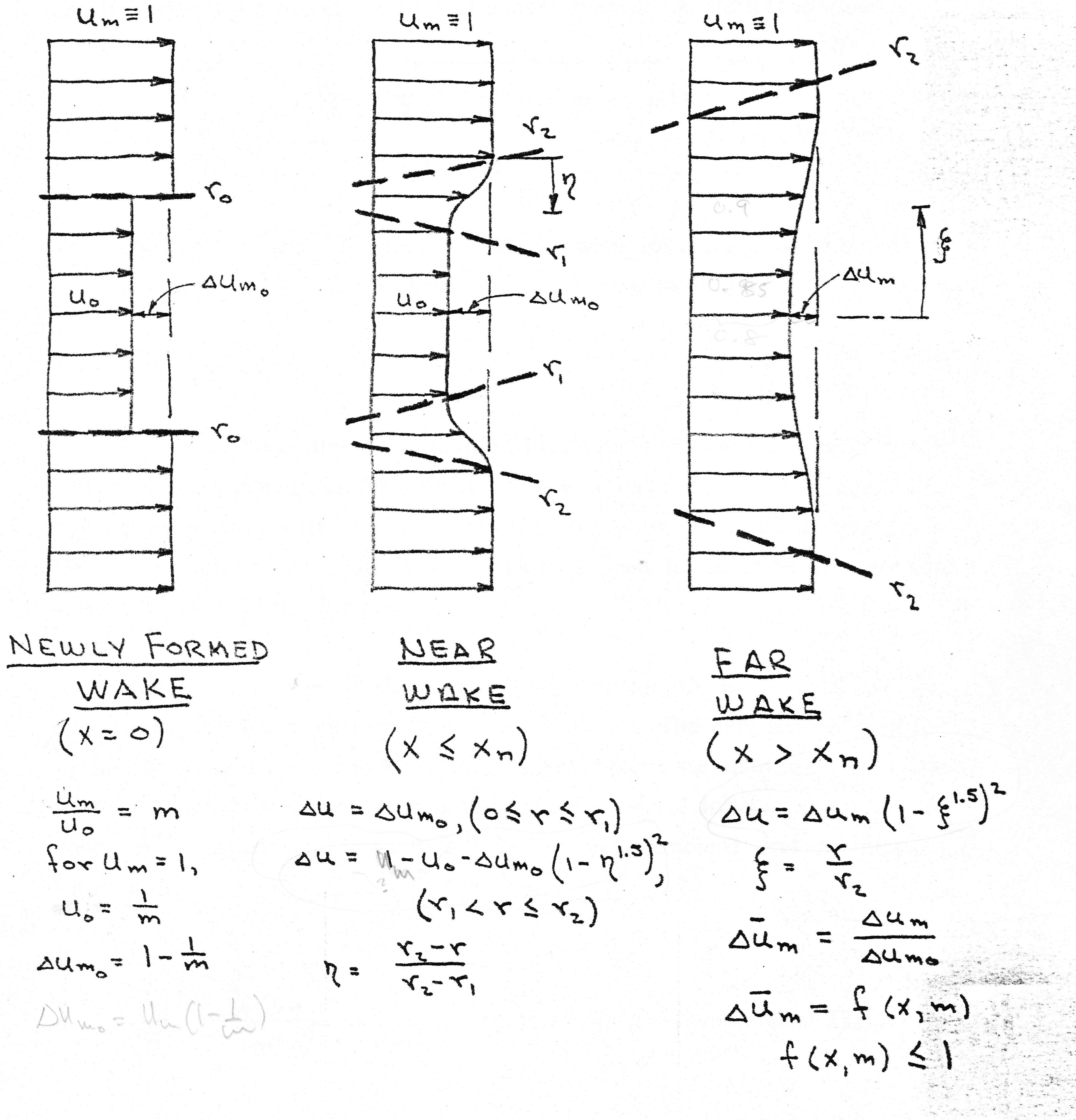


FIGURE 6. Basic wake velocity profiles.

quoted equation does not apply. However, for the fully developed wake profile outer wake radius for a given centerline velocity deficit is given by:

$$\Delta \bar{u}\, m = \frac{-.258m + \sqrt{(.258m)^2 + .536\,(1 - m)\left(\frac{r_0}{r_2}\right)^2}}{(.268)\,(1 - m)} \qquad (2)$$

This equation is derived from the streamwise invariance of momentum deficit or drag and is thus valid for any turbulent growth law.

### 3.3 Wake Growth

Figure 5 shows the wake divided into four separate regions in which different wake growth criteria exist. These criteria involve the basic fluid physics associated with the problem and have been discussed in Section 2. Here, we present the analytical expressions used to describe the growth criteria in each of the regions.

Region I is the initial region and its length is defined by the disappearance of the uniform flow core at $x = x_n$. The expressions used for $x_n$ are given below for the mechanical and ambient turbulence cases: The first is taken directly from Abramovitch; the second follows according to concepts discussed in Section 2 for combined ambient and momentum turbulence.

$$\frac{x_n}{r_0} = \frac{14\,(1 + m)}{(1 - m)\sqrt{1.49 + m}} \qquad \text{(momentum turbulence only)} \quad (3)$$

$$\frac{X_n}{r_0} = \frac{1}{\sqrt{\left(\frac{d}{.51}\right)^2 + \left[\frac{(.112 + .036m)\,(1-m)}{1+m}\right]^2}} \qquad \text{(momentum and ambient turbulence)} \quad (4)$$

The radii, $r_1$ and $r_2$ are given for this region as:

$$\frac{r_1}{r_0} = 1 + .22X \frac{1-m}{1+m} \left(a - .22\, bx \frac{1-m}{1+m}\right) \tag{5}$$

$$\frac{r_2}{r_0} = \frac{r_1}{r_0} - \frac{a - \sqrt{a^2 + 4b\left(1 - \frac{r_1}{r_0}\right)}}{2b} \tag{6}$$

$$a = .416 + .134\, m$$

$$b = (.021)(1 + .8\, m - .45\, m^2)$$

for the case of mechanical turbulence only and,

$$\frac{r_1}{r_0} = 1 - \frac{x}{x_n} \tag{7}$$

$$\frac{r_2}{r_0} = \left(1 + \frac{x}{x_n}\right)\left(\left(\frac{-a + \sqrt{a^2 + 4b}}{2a}\right) - 1\right) \tag{8}$$

for mechanical and ambient turbulence combined. This model incorporates linear development with x of both $r_2$ and $r_1$, a reasonable assumption for ambient turublence. Momentum conservation in this zone must be checked. The final radius, $r_{21}$, at the end of region I for both cases is given as:

$$\frac{r_{21}}{r_0} = \frac{-a + \sqrt{a^2 + 4b}}{2a} \tag{9}$$

The growth rate is this region is accordingly,

$$\left.\frac{dr}{dx}\right|_{I} = \frac{r_{21} - r_0}{x_n} \qquad (10)$$

Region II provides a transition between the velocity profiles of the near and far wake regimes. As yet, no analytical form exists for this transition, so for the present level of development of the code, the velocity profile associated with the far wake was used. For both the cases of mechanical and mechanical plus ambient turbulence, the radius, $r_{22}$, at the end of this region is:

$$\frac{r_{22}}{r_0} = \frac{1}{\sqrt{.134\,(1 - m) + .258\,m}} \,. \qquad (11)$$

The length of this region as proposed by Abramovitch (1963) is $x = 0.5\,x_n$ so that the growth rate here is:

$$\left.\frac{dr}{dx}\right|_{II} = \frac{r_{22} - r_{21}}{.5\,xn} \qquad (12)$$

For the case of mechanical turbulence only, the velocity deficit can be directly calculated by Equation 1 so since the radius of the wake is specified by the wake drag and the centerline velocity deficit, $\Delta um$, $r_2$ is given by

$$\frac{r_2}{r_0} = \frac{1}{\sqrt{(1-m)\,(.134\,\Delta\bar{u}m^2 + .258\,\frac{m}{1-m}\,\Delta\bar{u}\,m)}} \qquad (13)$$

where $\Delta\bar{u}\,m = \dfrac{\Delta um}{\Delta um_0}$ .

For the case of mechanical plus ambient turbulence, Region III represents the growth due to the influence of both types of turbulence and

Region IV represents the growth when the flow is dominated by ambient turbulence. Region IV extends downstream without bound, where the effects of ambient turbulence dominate, while Region III is defined to be $\Delta x = 10\ r_0$ in length since numerical calculations showed that momentum turbulence in the fully developed wake is virtually negligible after this distance.

The dependence of wake growth on both ambient and mechanical turbulence has been treated in Section 2 and the total growth derivative is expressed here as:

$$\left.\frac{dr}{dx}\right|_{III} = \sqrt{\left(\frac{\alpha}{.51}\right)^2 + \left(\frac{.22}{\left(\frac{m}{m-1}\right)\left(\frac{2}{\Delta \bar{u}\, m}\right) + 1}\right)^2}\ , \qquad (1)$$

where the value of $\Delta \bar{u}\, m$ is an explicit function of $r_2$ from equation (2). Thus, an integration must be performed to determine $r_2$ in this region. Figure 7 shows the results of such an integration for m = 3, corresponding to a power coefficient of 0.593, and various values of $\alpha$, the turbulent dispersion. The interesting feature noted in this figure is that wake growth is almost linear with distance for m values as high as m = 3. This indicates the dominance of the ambient term which is more pronounced for smaller power coefficients (or initial velocity ratio, m).

A series of numerical integrations for a range of m, $\alpha$ were performed and this linearity was quite evident. To illustrate this we define an effective dispersion $\alpha_e$, given by $\alpha_e = \Delta r / \Delta x$ in Region III. This effective dispersion then, is the combined wake growth of both ambient and momentum turbulence. The reduced results of the numerical integrations are shown in Figure 8. We note that at zero ambient turbulence ($\alpha$ =0), $\alpha_e$ is a strong function of momentum, represented by m. However, at larger values of $\alpha$ it is clear that $\alpha_e$ approaches the ambient turbulence level, indicating the dominant effect of this turbulence. How, it is unlikely that ambient turbulence under WECS operating conditions would be lower than $\alpha$ = 0.05. We note that even in this case, for a high power coefficient of $C_p$ = 0.50 (m = 2), the effective dispersion is only 9% higher than that due to ambient turbulence alone.

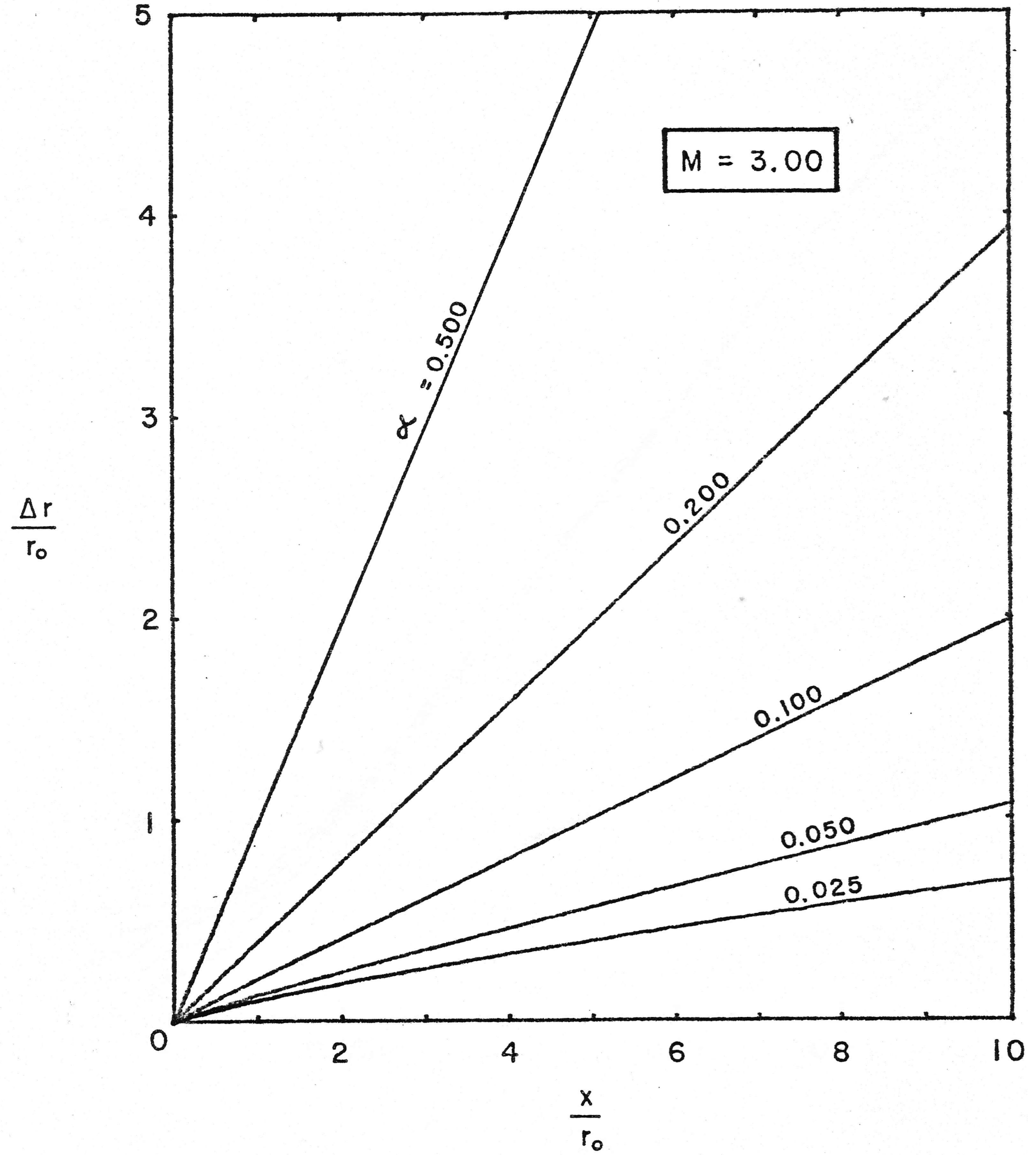


FIGURE 7. Wake growth due to varying ambient turbulence, α.

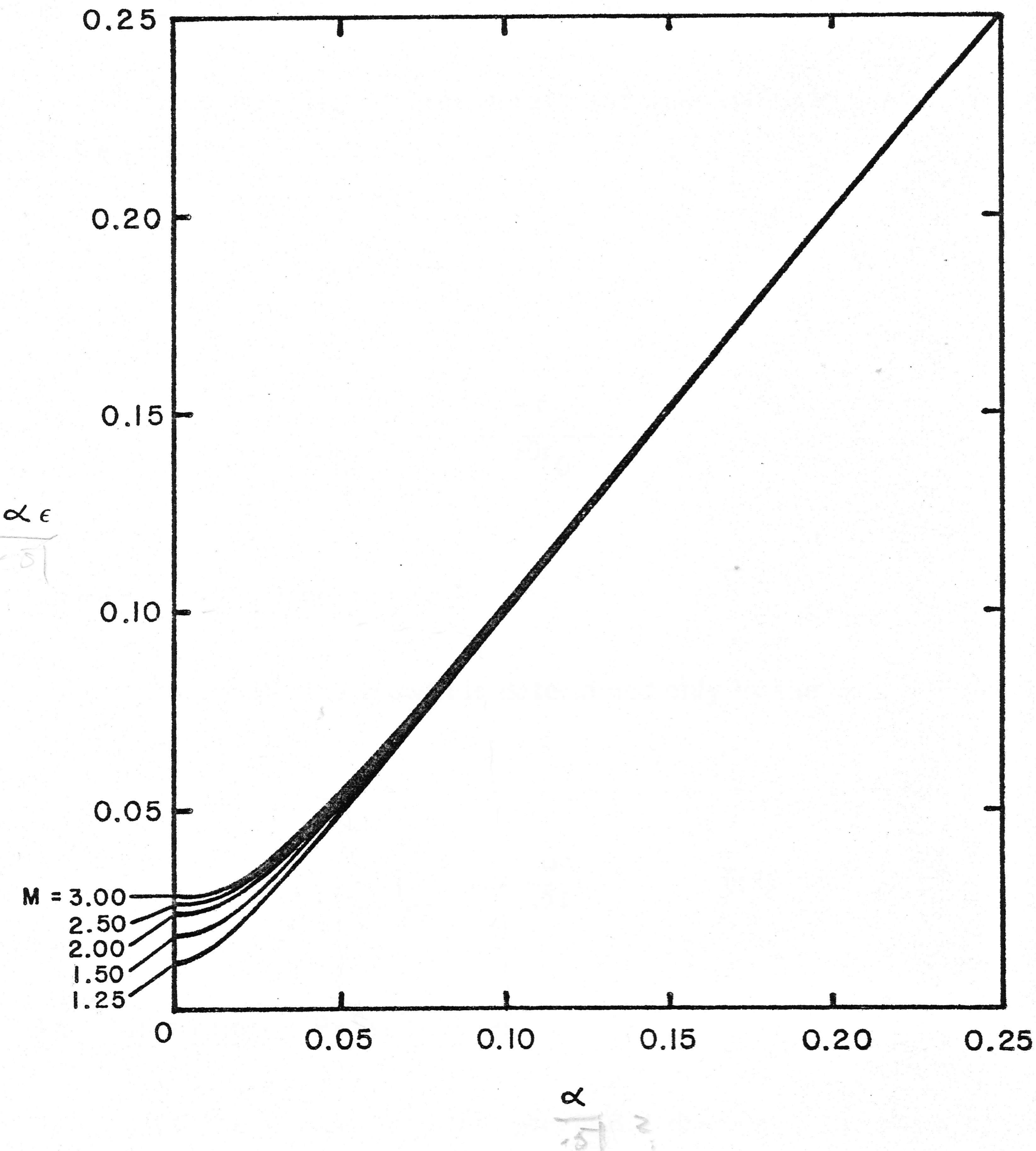


FIGURE 8. Effective dispersion, αε, as a function of turbulent dispersion, α, and initial velocity ratio, m.

The information on Figure 8 can be reduced to an algorithm for inclusion in the computer code, but at present the figure itself was used to generate the appropriate input for calculation of the dispersion and hence determination of wake growth in Region III.

Correspondingly, the growth rate and wake radius at the end of Region III are given by:

$$\frac{r_{23}}{r_0} = \frac{r_{22}}{r_0} + 10 \frac{\alpha}{.51} K, \qquad (15)$$

$$\left.\frac{dr}{dx}\right|_{III} = \frac{r_{23} - r_{22}}{10 r_0} , \qquad (16)$$

where K = $\alpha_e / \alpha$ from Figure 8.

In Region IV, the growth is determined only by the ambient turbulence and is given by:

$$\left.\frac{dr}{dx}\right|_{IV} = \frac{\alpha}{.51} \qquad (17)$$

## 3.4 Integration of Power Profiles

Using the velocity profiles previously discussed, the velocity at any downstream location may be found. Dividing this velocity by the free stream velocity and cubing the result gives the power flux ratio at that location, referenced to free stream conditions. By adding such power ratios incrementally over the area of the downwind disk, the total integrated, intercepted power is obtained for sufficiently small increments. Figure 9 shows the disk area and the approximation to this area that was used for the integration.

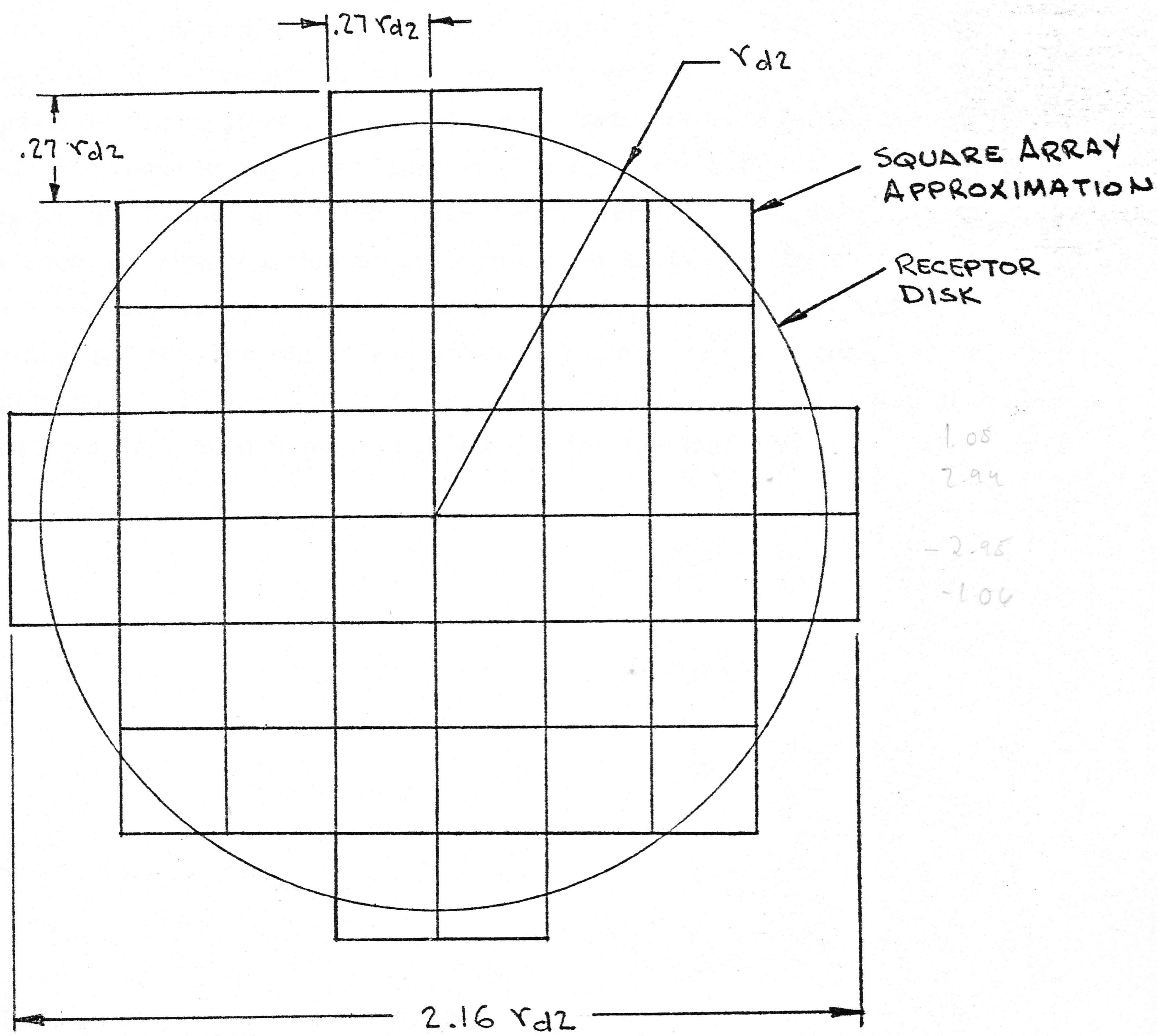


FIGURE 9. Receptor disc approximation for integration.

## 3.5 Effects of the Ground Plane

As previously discussed, standard image methods will adequately express the wake velocities when the influence of the ground is of importance. Figure 10 is a sketch of the geometry involved for the present model. The power at any given location is based on the total wake velocity. In regions where the source and image wakes overlap, the velocity deficits from each are simply added together and then subtracted from the free stream velocity to find the total wake velocity, which is then cubed to determine power. The source and image wakes are identical except for a geometrical offset in the height variable, z, corresponding to twice the height from the ground of the actual power extraction machine.

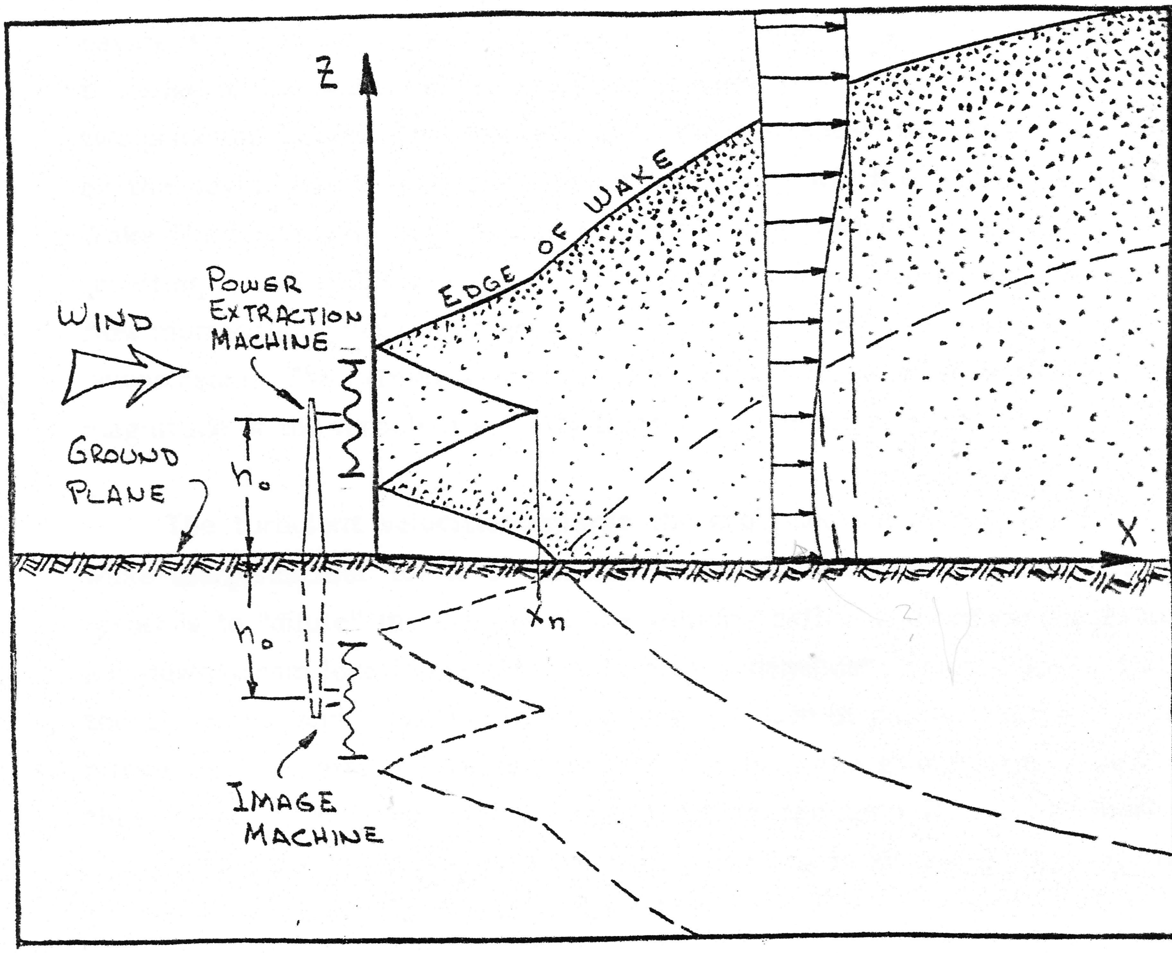


FIGURE 10. Image representation for the ground plane showing wake geometry.

# 4. AMBIENT TURBULENCE AND ITS RELATION TO WAKE GROWTH

The growth of the momentum deficit wake behind an energy extraction device is a function of the magnitude of the turbulent field which exists near the edge of the wake. As previously discussed, this turbulence arises from two principal sources, the mechanical (momentum shear) turbulence created by the device, itself, and the ambient turbulence which exists in the flow. Wake fluid is transported by the motions of the turbulent field, and the resulting outward diffusion of this fluid and inwards diffusion of the outer flow momentum gives rise to growth in the size of the wake as it progresses downstream. The rate at which this growth takes place is a function of the magnitude of the turbulent velocity fluctuations in the wake region.

The turbulent velocities enlarge the cross-section of the momentum wake (perpendicular to its centerline) in the downstream direction. The result is to "dilute" the effects of the velocity deficit and reduce shears at any downstream location resulting in a profile dependence much like that of the dispersion and reduction in local concentration of passive material in a plume released into a flowing stream. As the wake grows larger due to these turbulent motions, high energy fluid from the outer flow continuously mixes whith the lower energy wake fluid, resulting in an asymptotic return to free stream conditions at some downstream location.

Under the special conditions of a placid (zero ambient turbulence) wind, wake growth is a result only of mechanically generated turbulence. This turbulence is created by the energy extraction device, itself, during the energy extraction process. The magnitude of the resulting turbulent velocities is related to the amount of power extracted from the fluid, and can be expressed in terms of the velocity ratio, m.

Mechanical turbulence is contained wholly within the wake itself, by the same fluid particles which carry the momentum deficit. As the wake expands under the action of turbulent diffusion, the density of the turbulent energy reduces, resulting in a "dilution" of its effects on further diffusion.

Thus, for the case of mechanical turbulence only, the diffusion rate of the wake slows with increasing downstream distance. Accordingly, the wakes grow slowly and they can retain a large amount of momentum deficit over long downstream distances.

The situation is changed dramatically when ambient turbulence is added to the flow. Here, turbulence is everywhere present in the fluid field to continue the diffusion process at a rate fixed by the level of this turbulence. As the wake grows larger, the turbulent energy for further diffusion does not decrease but rather stays constant, so that the wake growth rate remains high. Thus, the energy recovery is much more rapid than that which would occur due to mechanical turbulence alone (zero ambient turbulence). The power available to a second machine contained in the wake when ambient turbulence is taken into account correspondingly increases for higher ambient turbulence levels.

The situation most often encountered in the atmosphere at moderate wind speeds is for significant ambient turbulence to be present. Wake geometry and flow conditions are thus dominated by the level of ambient turbulence for otherwise fixed conditions, i.e , size and power, etc. of the upstream energy extracting device. Although the mechanical turbulence is always an omnipresent factor on wake development (particularly in the early stages), the effects of ambient turbulence usually control the downstream growth.

The level of ambient turbulence in the surrounding flow is determined by the action of three separate effects; wind speed, ground roughness, and vertical atmospheric motions due to convection produced by solar heating. The space and time dependent nature of each of these variables is normally taken into consideration by evaluating them over a range in upstream distance from the wake-producing device. Thus, the level of ambient turbulence is known for the flow as it enters the region of interest. Any changes in downstream flow conditions in the wake region produce effects that are sufficently small to be ignored. The effects of turublent wake transport, then, are wholly determined by the appropriate conditions upstream of the wake generator. This is the situation for which the present model was developed.

The wind speed determines the amount of shear-produced turbulence in the boundary layer flow near the ground. Often, this is the principal source of ambient turbulence for flows over flat, level regions. Both ground roughness and vertical thermal convection create turbulent fluctuations in the moving air mass. Under conditions of unstable or neutral atmospheric stratification, these fluctuations become exaggerated and distorted, and generally decompose into additional components of turbulence. Under stable atmospheric conditions, with light or low winds, normally <u>all</u> turbulence components (including the wind shear generated ones) are suppressed, corresponding to flows above the critical Richardson's Number.

Thus, the ambient turbulence level (and hence the dispersive qualities) of the flow into the energy extraction device and its wake region is a combination of the upstream meteorological conditions and terrain. A standard procedure for treating diffusion problems under such circumstances is to create categories for the various measurable parameters over which prescribed variations in these parameters can occur. These categories are then grouped in various ways into classes. Diffusion coefficents for the flow are developed and experimentally correlated under field conditions for which the different classes obtain. These results are well-known (Pasquill 1962), and have been quite useful in characterizing dispersive flows in the atmosphere.

In recent work at AeroVironment by Lissaman (1973), the effects of ground roughness and heat flow were explicitly included in the analytical form of the diffusion relationships, obviating the need for the Pasquill classification system. Validation of the AeroVironment model has shown excellent correlation with field measurements (AeroVironment staff, 1972).

# 5. FUTURE RESEARCH PLANS

## 5.1 General

It will be seen that the work so far has satisfactorily met the goals of Phase I, and in some respects made more progress than initially estimated. A computer model has been developed, programmed and run which takes into account most of the significant parameters relating to WECS. Two parameters need further study. These are the effects of the ground plane in suppressing ambient turbulence close to the ground and the effects of a non-uniform incoming wind profile. Some fluid dynamical aspects of these situations are discussed in more detail in further sections.

The major task for Phase II will be essentially as described in the proposal AV SP 637. This is to develop a general model for the wakes of WECS arrays, and to exercise this model with different parameters to identify the sensitivity to various terms. The present research has already shown that ambient turbulence is a dominating factor in the wake development and the realization of this introduces quite significant simplifications into the modeling.

Additionally, it is still necessary to complete the literature search for any further references of value, or any apparently unrelated work which may provide data for the current project. This is discussed in more detail in the following paragraphs.

## 5.2 Effect of Ground Plane

We have already made a first order incorporation of the ground plane in the current model by using standard imaging techniques. As has been shown previously, the use of techniques associated with passive plume development is likely to be accurate and effective here, because the turbulence generation due to momentum is small compared with ambient turbulence. Thus the techniques used to handle plumes in ground effect should be of value here, and should form a good basis for either validating the simple model already being used, or for improving it. As noted, standard U.S. procedures are to model the ground with a direct image (Slade, 1968). However, rational analytical procedures for the ground effect and shear flows are extensively described by Monin and Yaglom (1971). It is felt that a careful check of the validity of image modeling must be made, and if more accurate procedures are available, these will be incorporated into the model.

## 5.3 Effect of Non-Uniform Incoming Wind

Thus far the analysis has concentrated upon the wakes of rotors immersed in a wind field of uniform velocity and turbulence. This is certainly an excellent first order approach, however, account must be taken of non-uniformities in the wind and incoming turbulence. These occur through two influences; first, due to the natural planetary boundary layer profile where the wind increases with vertical height and, second, due to the presence of upstream rotor systems which will produce disturbances in the incoming field.

For the planetary boundary layer there exists a large amount of data on wind and turbulence profiles, for example Lumley and Panofsky (1964). Techniques of relating this to ground roughness and insolation are quite well-understood. The principal theoretical task for this project will be to model the dynamics of wake growth in this non-uniform profile. Here new expressions for the drag integral in a non-uniform flow must be developed, as well as expressions for local growth rate of the turbulent wake. Work is in progress on this formulation. One simple model being studied is to consider the wind profile to be stepwise discontinuous at the axis of the rotor, so that the upper portion is immersed in one flow of uniform speed and turbulence, and the lower portion of the rotor in a different but also uniform field. Such a model will not involve any additional fluid dynamic analysis and require only additional standard routines in the existing computer program. An extensive study of this approach, and comparison with more sophisticated procedures, will be made, the goal being to develop an effective accurate model. It is expected that a sensitivity analysis, to quantify the actual magnitude of the non-uniformities and their effect will reveal that a simplified approach will be adequate.

The second aspect of non-uniform incoming flow is the disturbance due to upstream rotor systems. Development of a model for this will follow the study of the effect of the natural wind non-uniformity. For the case of disturbances due to upstream rotors, it is expected that the velocity differences will be quite small, and also that the rotor induced turbulence will be small compared with the ambient, as has already been shown in the research done to date.

## 5.4 Literature Search

It was felt important to study the actual fluid physics of the situation at the beginning of the project, and to develop a first model to identify the complexity and detail required in any analytical or computer model. Thus, the current computer models were constructed using best theoretical and experimental data available, with the point of view that if further research indicated adjustments to the model constants should be made, that these could be rapidly and simply incorporated into the program code.

Thus is was felt expedient to start programming the model at the earliest opportunity, before the literature search was complete. This has proved a sound decision, since the literature search is now much more closely defined.

Most of the major published volumes on jets and wakes have already been consulted in the search but only those directly utilized here are included in the attached reference list. An exhaustive literature search is now in progress, specifically aimed at the problem of momentum diffusion in a turbulent non-uniform medium.

Much of the current knowledge is, in fact, summarized and reviewed in two modern books; Tennekes and Lumley (1972) and Monin and Yaglom

(1971), while an excellent engineering approach has been given by Abramovitch (1963). This latter approach has formed the basis of our models.

Although there is an extensive body of both experimental and theoretical research on wakes and jets, the majority of this work relates to non-turbulent outer flows, a situation which, while relevant to our work, does not take into account the very important effect of ambient turbulence which dominates wake development in the natural wind. The problem of outer flow turbulence is briefly discussed by Abramovitch. He shows that, for small outer turbulence of intensity of about 0.5% of the external mean flow, that the wake decay is greatly accelerated. Abramovitch proposes asymptotic formulae for zero outer turbulence and for dominant outer turbulence, which do, in fact, limit to our more general expression.

Recent research which contains very up-to-date experimental work on wake and jet development is contained in an unpublished thesis by Higuchi (1977). Although this work does not include outer turbulence, some of the excellent experimental data on internal jet turbulence will probably be of importance in validating our models. This research was part of a Ph.D. thesis completed at the California Institute of Technology, and AeroVironment researchers are currently in communication with Dr. Higuchi's thesis supervisor.

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